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LANDING AND BRAKING OF AIRPLANES

By Louis Breguet

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The maneuver of landing on a normal field with a good airplane presents no danger for a capable and prudent pilot. In the event of a forced landing, however, a pilot may be compelled to alight on a field more or less unsuitable as to smoothness, size, and orientation. In such a case it is obvious that the landing may be difficult, if not dangerous.

Various devices have been proposed for overcoming these difficulties, which will eventually disappear of themselves with a suitable distribution of the engine-propeller groups. Airplanes have been made with lifting propellers for taking off and landing vertically. Such airplanes, however, have poor aerodynamic qualities and are exposed to very great danger, if their lifting propellers stop or fail to function properly.

As for normal airplanes, it has been sought, in particular, to reduce their minimum flight speed, which is regarded as their practical landing speed. Perhaps too much stress has been laid on reduction of landing speed, which obviously constitutes an important element of safety, but which should not be considered exclusively. Other elements, such as the angle of glide and the shortening of the landing run by air and ground brakes, have an

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important bearing on the safety of landing.

We propose to treat briefly the various aspects of this question. We shall, however, consider only land airplanes, because the case of seaplanes is quite different. For the latter, the area for alighting is seldom restricted or surrounded by obstacles.

Landing.- Landing maneuvers may be divided into three phases:

1. Gliding descent;
2. Levelling off near the ground;
3. Making contact with the ground, taxying and stopping.

We will analyze these three phases.

Gliding descent.- In this phase the gas and ignition are switched off, and the propeller functions as a windmill and adds a certain structural drag to the drag of the airplane proper.

Let π represent the polar curve of the airplane (Fig. 1) under these conditions. Let M denote the point on this curve corresponding to the regime considered; P, the weight of the airplane; and S, the wing area. The flight speed V is then determined by formula

$$V = \sqrt{\frac{P}{\frac{\rho}{2} S \, \overline{OM}}},$$

the segment $OM = \sqrt{c_x^2 + c_z^2}$ being, moreover, practically equal, on the polar, to the segment $\overline{OM} = c_z$.

The inclination θ of the flight path is equal to the angle φ which OM makes with the axis c_z. On designating the lift-drag ratio (c_x/c_z) at the regime considered by $\tan \varphi$, it is obvious that

$$\tan \theta = \tan \varphi = \frac{c_x}{c_z}.$$

Lastly, the vertical speed of descent b equals $V \sin \theta$. V, θ and v are three important elements in gliding flight. When a pilot sights his landing field the accuracy of his course is improved in proportion to his angle of glide θ. Moreover, this enables him to clear an object of height H (Fig. 2) at the edge of the field by directing his course toward a point A nearer the obstacle, since $D = \frac{H}{\tan \theta}$.

Simultaneously with the increased angle of glide θ, it is obviously advantageous to reduce the flight speed V or the vertical speed of descent $(v = V \sin \theta)$.*

There are four distinct regimes of gliding descent.

1. Regime of minimum speed V.- This corresponds to the point M_1 for which OM is the maximum. This point is found a little to the right of the point M of the polar (Fig. 3) for which c_z is the maximum. In practice it is necessary to

*It cannot be denied that the reduction of the minimum angle of glide $\theta_s = \varphi_s$ is also of considerable importance. If, in cross-country flight, engine trouble develops at a certain altitude, the farthest points attainable by gliding are at a distance proportional to $\cot \theta_s$, and the area within which the pilot can seek a propitious landing field is proportional to $(\cot \theta_s)^2$. While trying to increase the maximum angle of glide, we must also try to reduce the minimum angle of glide as much as possible.

disregard the portion of the polar situated to the right of M_2. We shall therefore replace M_1 by M_2, the difference in the corresponding speeds being negligible.

2. Regime of maximum slope θ.- The descent can be made at 90°, corresponding to the points M_3 or M_3'. The point M_3 corresponds to an impossible regime (descent in the horizontal position) in which the airplane possesses neither stability of flight nor efficacy of the controls. The point M_3' corresponds to a vertical dive, i.e., to an extremely swift regime. Practically, in order to descend at the slow regime and at the maximum slope, it is necessary to fly at the regime M_2, the useful limit of the polar corresponding precisely, from what precedes, to the practical minimum of the speed V. It would not be prudent, however, to fly at this regime, since the airplane would then have no protection, i.e., no sufficient margin of control.

3. Regime of minimum vertical speed v.- Here we have

$$v = V \sin\theta = \sqrt{\frac{P}{\frac{\rho}{2} S}} \; \frac{c_x}{(c_z)^{3/2}} \; \frac{1}{(1 + \tan^2\varphi)^{\cancel{3/4}}} \; {}^{1/2}$$

Taking $1 + \tan^2\varphi$ equal to unity, which is practically true for the regimes considered, the minimum of v corresponds to the minimum of $c_x/c_z^{3/2}$, i.e., at the regime of minimum power necessary for horizontal flight. The corresponding point M_4 is therefore necessarily situated between the point M_2 (maximum of c_z or minimum of V) and the point M_5 corresponding to the minimum coefficient of glide $\tan\varphi$ and consequently to the minimum angle of glide θ.

4. Regime of minimum slope (or angle of glide).- This is the regime M_5 corresponding to the minimum coefficient of glide (or L/D ratio) $\tan\varphi$.

Let us see how the various regimes are affected by modifications of the polar curve π. For existing airplanes, the polar curve π, between the points M_5 and M_2, can be assumed to be a portion of a parabola, whose axis coincides with the axis of the abscissas c_x and of the equation

$$c_x = a + b\, c_z^2$$

The limiting ordinate of this parabola is c_{z_M}, which can be taken as coinciding with the point M_2 of the real ordinate c_{z_M} situated by hypothesis beyond M_4. By always taking $1 + \tan^2\varphi$ equal to unity, for simplicity, it is easily seen that the principal regimes, as defined above, are characterized as follows.

TABLE I.

Regime	c_z	$\tan\varphi = \tan\theta$
Regime M_5 (minimum of θ)	$c_{z_5} = \sqrt{\frac{a}{b}}$	$\tan\theta_5 = 2\sqrt{ab}$
Regime M_4 (minimum of v)	$c_{z_4} = \sqrt{3}\, c_{z_5}$	$\tan\theta_4 = 1.16 \tan\theta_5$
Regime M_2 (minimum of V and maximum of θ)	$c_{z_2} > c_{z_4}$	$\tan\theta_2 > \tan\theta_4$ $\theta_2 = \theta_{max}$

TABLE I (Cont.)

Regime	V	$v = V \sin\theta$
Regime M_5 (minimum of θ)	$V_5 = K\sqrt{\frac{b}{a}}$	$v_5 = 2\,K\,b$
Regime M_4 (minimum of v)	$V_4 = 0.76\,V$	$v_4 = 0.88\,v_5$
Regime M_2 (minimum of V and maximum of θ)	$V_2 < V_4$ $V_2 = V_{min}$	$v = v_4\sqrt{\frac{c_{z_4}}{c_{z_2}}}$

Let us consider another polar, likewise constituting a parabola for the region considered. It can be derived from the preceding parabola by two successive transformations, a homothety with respect to the point O and a translation parallel to the axis of the abscissas c_x. We shall examine these two transformations successively.

a) Homothetic transformation.- The arc $M_5\,M_2$ of the parabola π is transformed into the arc $M_5{}'\,M_2{}'$, homothetic with respect to O, in the ratio λ of the parabola π'. For this new polar, of the equation

$$c_x' = a\lambda + \frac{b}{\lambda} c_x^{2\prime}$$

the regimes of gliding flight are characterized as follows.

TABLE II.

Regime	c_z	$\tan\varphi = \tan\theta$
Regime M_5' (minimum of θ)	$c_{z_5}' = \lambda\, c_{z_5}$	$\tan\theta_5' = \tan\theta_5$
Regime M_4' (minimum of v)	$c_{z_4}' = \lambda\, c_{z_4}$	$\tan\theta_4' = \tan\theta_4$
Regime M_2' (practical min. of V and max. of θ)	$c_{z_2}' = \lambda\, c_{z_2}$	$\tan\theta_2' = \tan\theta_2$

TABLE II (Cont.)

Regime	V	$v = V \sin\theta$
Regime M_5' (minimum of θ)	$V_5' = \frac{V_5}{\sqrt{\lambda}}$	$v_5' = \frac{v_5}{\sqrt{\lambda}}$
Regime M_4' (minimum of v)	$V_4' = \frac{V_4}{\sqrt{\lambda}}$	$v_4' = \frac{v_4}{\sqrt{\lambda}}$
Regime M_2' (practical min. of V and max. of θ)	$V_2' = \frac{V_2}{\sqrt{\lambda}}$	$v_2' = \frac{v_2}{\sqrt{\lambda}}$

b) Translation of polar.- The arc $M_5\ M_2$ of the parabola π is transformed into the arc $M_5''\ M_2''$ of the parabola π'' (Fig. 4) following the axis c_x and separated from the former by a constant distance $\Delta a = a'' - a$. For this new polar, of the equation

$$c_x'' = a'' + b\, c_z^{2}{}'',$$

the regimes of gliding flight are characterized as follows.

TABLE III.

Regime	c_z	$\tan\varphi = \tan\theta$
Regime M_5'' (minimum of θ)	$c_{z_5}'' = c_{z_5}\sqrt{\frac{a''}{a}}$	$\tan\theta_5'' = \tan\theta_5\sqrt{\frac{a''}{a}}$
Regime M_4'' (minimum of v)	$c_{z_4}'' = c_{z_4}\sqrt{\frac{a''}{a}}$	$\tan\theta_4'' = \tan\theta_4\sqrt{\frac{a''}{a}}$
Regime M_2'' (practical min. of V and max. of θ)	$c_{z_2}'' = c_{z_2}$	

TABLE III (Cont.)

Regime	V	$v = V \sin\theta$
Regime M_5'' (minimum of θ)	$V_5'' = V_5\sqrt{\frac{a}{a''}}$	$v_5'' = v_5$
Regime M_4'' (minimum of v)	$V_4'' = V_4\sqrt{\frac{a}{a''}}$	$v_4'' = v_4$
Regime M_2'' (practical min. of V and max. of θ)	$V_2'' = V_2$	

Every parabolic polar can be derived from another by a homothety followed by a translation. The preceding tables show the resulting modifications for the principal regimes considered above.

In particular, if we consider the regime of minimum vertical speed, which practically coincides with the regime of best climb, we see that the homothety reduces the speed (V_4 and v_4 becoming V_4' and v_4') without altering the descending slope, while the translation changes both the slope and the flight speed without affecting the vertical speed (Fig. 5).

The deformation of a polar by homothety closely approximates the result obtained by providing a wing with suitably disposed slots and auxiliary ailerons. The deformation of a polar by translation parallel to the c_x axis is the one obtained by adding the structural drag or by causing the propeller to act as a brake in such manner as to produce a braking effect proportional to the square of the speed, which is obtained by varying the revolution speed of the braking propeller in proportion to the speed of the airplane.

An airplane necessarily glides at a regime appreciably swifter than that of maximum lift, both to preserve a suitable controllability and to maintain a sufficient momentum for levelling off near the ground. In practice it has been found necessary to glide at about 2/3 of the maximum lift. For an ordinary airplane, this corresponds closely to the regime M_4, i.e., to the regime of minimum vertical speed.

Therefore the homothetic transformation of the polar enables the reduction of V_4 or v_4 without altering the corresponding angle of glide. The translation of the polar makes it necessary to maintain the value $c_z = c_{z_4}$ of the lift, which represents the chosen fraction of the maximum lift. In this case the tangent of the angle of glide is increased by the quantity

$$\Delta \tan\theta = \frac{\Delta a}{c_{z_4}} = \frac{a'' - a}{c_{z_4}}$$

without appreciably affecting the flight speed.

As an illustration, let us consider an airplane descending in the regime M_4 at an angle of 5° ($\tan\theta_4 = 0.1$) with a speed V_4 = 114 km/h (70 mi./hr.), P/S = 50 kg/m² (10 lb./sq.ft.), $c_{z_4} = 0.8$, i.e., with a vertical speed of v_4 = 3.2 m (10.5 ft.) per second. Table IV shows the effect of the various possible modifications of the polar of this airplane on the angle of descent θ, on the flight speed V, on the vertical speed v and on the horizontal distance D traversed in descending a vertical distance of H = 50 m (164 ft.).

TABLE IV.

Modifications made in the airplane	Regime of gliding flight c_z = constant fraction of max. lift				
	θ	$\tan\theta$	V km/h	$v=V \sin\theta$ m/s	$D=H/\tan\theta$ (H=50 m)
a) Reference airplane	5°40'	0.10	114	3.2	500
b) Structural drag represented by $\Delta c_x=0.04$ practically doubling min. drag of airplane.	8°30'	0.15	113	4.7	333
c) Braking effect of propeller represented by $\Delta c_x=0.16$.	16° 0'	0.30	111.5	8.9	166
d) Sum of b + c	19°20'	0.35	111.0	10.2	143
e) Homothetic transformation of the polar in the ratio $\lambda = 2$.	5°40'	0.10	80	2.26	500
f) Sum of b + c + e.	19°20'	0.35	78	6.25	143

This table, given only by way of illustration, shows the effect of the methods of increasing the head resistance or drag of the air (by means of supplementing retarding devices or by the propeller) on the reduction of D and the effect of the methods of amplifying the polar (by slotted wings, for example) on the reduction of V and v. It is important to reduce not only V and v, but also D. It is obviously desirable to devise, if possible, a judicious combination of these various methods of modifying the polar in gliding flight.

We must not forget, however, that it is desirable to increase the L/D ratio at the regime of gliding flight for landing, which can be accomplished only by methods incapable of impairing the L/D ratio at the regime of normal flight. Though this condition is very important for maintaining the excellence of an airplane in normal flight, it is rather difficult to satisfy.

Levelling off preparatory to landing.- This phase extends from the moment the pilot abandons, near the ground, the regular regime of gliding flight up to the moment the airplane touches the ground. In order to describe the corresponding flight path AB (Fig. 6), the pilot gradually noses up his airplanes, which reduces the angle of descent and the speed. At any instant, on projecting the acceleration of the center of gravity of the airplane and the external forces on the speed and its normal, we have

$$\frac{P}{g}\,\frac{dV}{dt} = P\sin\theta - c_x\,\frac{\rho}{2}\,SV^2, \qquad (1)$$

$$\frac{P}{g}\,\frac{V^2}{R} = c_z\,\frac{\rho}{2}\,SV^2 - P\cos\theta\,. \qquad (2)$$

Equation (2), in which R (the radius of curvature) is positive, shows that

$$c_z\,\frac{\rho}{2}\,SV^2 > P\cos\theta\,,$$

i.e., that the speed is greater than that of regular gliding flight for the lift c_z. From this inequality we deduce

$$c_x\,\frac{\rho}{2}\,SV^2 = \tan\varphi\;c_z\,\frac{\rho}{2}\,SV^2 > P\cos\theta\,\tan\varphi = P\sin\theta\,\frac{\tan\varphi}{\tan\theta}$$

The ratio $\tan\varphi/\tan\theta$, equal to 1 at the point A, keeps increasing along AB, because $\tan\varphi$ increases (due to the gradual levelling off of the airplane) and $\tan\theta$ decreases. As a result

$$c_x \frac{\rho}{2} S V^2 > P \sin\theta$$

and equation (1) shows that dV/dt is negative, i.e., that the speed V goes on diminishing.

The whole art of piloting, for a correct landing, consists in levelling off gradually along AB, so that the flight path is tangent to the ground at the point B and so that, at this point:

a) the lift c_z is practically maximum;

b) the curvature $(1/R)$ of the flight path is practically zero, these two conditions serving to render the speed V_B at the point B as near its minimum as possible;

c) a slight supplementary mosing-up of the airplane, effected by pulling rather abruptly on the control stick, causes it to touch the ground with the tail skid or simultaneously with the tail skid and wheels in such manner as to avoid any rebounding of the airplane from the ground.

In order to realize the path AB, it is obviously necessary to have sufficient reserve lifting power at the point A. For this reason, as we have already mentioned, a regime of gliding flight considerably below that of the maximum lift must be adopted.

The complete solution by calculation is too difficult to be attempted here. Moreover, the skill and ability of the pilot greatly affect the length of the flight path L and of the drop h during the process of levelling off on the arc AB. We shall therefore content ourselves with assigning approximate values to these quantities, simply to enable us to estimate the effect of varying the essential characteristics of the airplane.

Let V_1 and V_2 represent the speeds at A and B and let θ_1 represent the initial angle of glide at A. By applying the momentum theorem to the airplane on the path AB, we obtain

$$\frac{P}{g}\,\frac{V_2^2 - V_1^2}{2} = Ph - P\left[\tan\varphi\left(\cos\theta + \frac{1}{g}\,\frac{V^2}{R}\right)\right]L_{\text{mean}} . \qquad (3)$$

This equation, involving a certain mean value of the function $\tan\varphi\left(\cos\theta + \frac{1}{g}\,\frac{V^2}{R}\right)$, renders it possible to link L and h to the initial and final speeds V_1 and V_2. In order to calculate L or h, it would be necessary to establish a second relation between them. For simplicity, let us assume that the arc AB (Fig. 7) is comparable to the arc of a circle. Under these conditions

$$h = D \tan\frac{\theta_1}{2}$$

and, since $\theta_1/2$ is always very small,

$$L = D = \frac{2\,h}{\tan\theta_1}$$

On the other hand, we have assumed that one flies, in a gliding descent, at a lift c_{z_1} which represents a certain fraction k of the maximum lift c_{z_M} ($k < 1$ and, for example, $k = 0.65$). Let us assume that the lift is the greatest ($c_{z_2} = c_{z_M}$) at the point B and that the lifting force exactly balances the weight, i.e., that the real curvature of the flight path becomes zero. Under these conditions

$$c_{z_2} = c_{z_M} = \frac{c_{z_1}}{k}$$

and, considering $\cos\theta_1$ as unity,

$$V_1 = V_2 \frac{c_{z_2}}{c_{z_1}} = \frac{1}{K} V_2^2. \qquad (5)$$

On taking equations (4) and (5) into account, equation (3) enables us to calculate L or D and h, obtaining

$$L = D = \frac{2}{\tan\theta_1} h = \frac{2}{\tan\theta_1} \frac{V_2}{2g} \frac{\left(\frac{1}{K} - 1\right)}{\left[\frac{2}{\tan\theta_1}\left[\tan\varphi\left(\cos\theta + \frac{1}{g}\ \frac{V^2}{R}\right)\right] - 1\right]_{mean}}$$

If the above-defined levelling-off path AB of a given airplane is known, it is easy to see how it is affected by changes in the airplane polar.

a) Let us assume a homothetic transformation of the polar in the ratio λ. The flight paths are then similar in the ratio of the squares of the speeds at the homologous points, i.e., in the ratio $1/\lambda$, and we have $h' = h/\lambda$ and $D' = D/\lambda$.

b) Let us assume a translation of the polar parallel to the c_x axis and equal to $a'' - a$. Starting from the same point, the two airplanes will have the same speed at the same level, if they fly at lifts c_z and c_z'' such that $c_x/\sin\theta = c_x''/\sin\theta''$. After descending from the height h, their respective flight paths will have a zero slope $(\theta_2'' = \theta_2 = 0)$, the same speed and an infinite radius of curvature. It is obvious, therefore, that the height h, of the levelling-off path is the same for both airplanes and that the corresponding horizontal distances D and D" are such that

$$D'' = \frac{D}{1 + \left(\frac{a'' - a}{c_x''}\right)_{mean}} .$$

Let us apply these results to the above numerical example. For the chosen reference airplane, we may assume that V_2 = 91 km/h (56.6 mi./hr.), h = 9 m (29.5 ft.), and D = 180 m (590 ft.). Table V is based on the same hypotheses as Table IV.

TABLE V

Modifications made in the airplane	Levelling-off characteristics		
	Slope at origin $\tan \theta_1 =$	Speed V_1 at origin km/h	Speed V_2 on landing km/h
a) Reference airplane	0.10	114	91
b) Structural drag represented by $\Delta c_x = 0.04$ practically doubling minimum drag of airplane	0.15	113	91
c) Braking effect of propeller represented by $\Delta c_x = 0.16$	0.30	111.5	91
d) Sum of b + c	0.35	111	91
e) Homothetic transformation of the polar in the ratio $\lambda = 2$	0.10	80	64
f) Sum of b + c + e	0.35	78	64

TABLE V (Cont.)

Modifications made in the airplane	Levelling off characteristics	
	Height in meters	Horizontal distance D in meters
a) Reference airplane	9	180
b) Structural drag represented by $\Delta c_x = 0.04$ practically doubling minimum drag of airplane	9	143
c) Braking effect of propeller represented by $\Delta c_x = 0.16$	9	113
d) Sum of b + c	9	110
e) Homothetic transformation of the polar in the ratio $\lambda = 2$	4.5	90
f) Sum of b + c + e	4.5	55

This table shows how the use of braking devices shortens the horizontal length of the flight path during the levelling off and how the amplification of the polar reduces the height and the horizontal length of this phase.

Of course it is necessary to consider the phase of gliding descent preceding the levelling off, when there is occasion to clear an obstacle whose height H is greater than the height h of the levelling-off phase. For example, let us suppose that one wishes to clear an obstacle of height H = 30 m (98 ft.). For the horizontal distance D netween the obstacle and the point of contact with the ground, he then finds the following values:

Case	a	$D = 390$ m
"	b	260 "
"	c	183 "
"	d	170 "
"	e	345 "
"	f	128 "

Of course these horizontal distances are determined on the supposition that there is no wind and that the flight path is entirely included in a given vertical plane.

By landing in a head wind or by describing a curving flight path on both sides of a mean vertical plane, the pilot can reduce the distance D, but the horizontal projection of the actual flight path remains practically equivalent to the above figures, thus demonstrating the importance of braking an airplane in the air, in order to land in a limited space bordered by obstacles.

Braking an airplane on the ground.- After an airplane has landed, it has the speed V^2 and is supported chiefly by its wheels and tail skid. The problem of braking on the ground then consists in stopping the airplane within as short a distance as possible.

The forces acting on the airplane are: the weight P applied at the center of gravity G; the total aerodynamic reaction R, of components R_x, R_z and lever arms δ with reference to G; the braking force F of the propeller, practically horizontal with lever arms d with reference to G; the normal reaction N_B and the tangential reaction $f\ N_B$ of the ground on the skid; the normal reaction N_R and the tangential reaction $\mu\ N_R$ of the ground on the wheels (neglecting the couple P of the rolling friction of the wheels on the ground). The positive directions of the various forces are indicated by the arrows in Figure 8. The equations for the horizontal motion of the center of gravity are:

$$\frac{P}{g}\ \frac{dV}{dt} = F - R_x - \mu\ N_R - f\ N_B \tag{1}$$

$$0 = R_z + N_R + N_B - P, \tag{2}$$

$$0 = Fd - R\delta + (a - \mu H)\ N_R - (b + fH)\ N_B \tag{3}$$

In order to verify equation (3), that is, for the skid to touch the ground $(N_B > 0)$, it is obvious that the aerodynamic couple $R\delta$ must have a suitable value, which the pilot obtains by a suitable deflection of the elevator, We then have:

$$N_B = \frac{Fd - R\,\delta}{L + (f - \mu)\ H} + (P - R_z)\ \frac{a - \mu\,H}{L + (f - \mu)\ H}$$

We will assume that the skid remains in contact with the ground, that is, that N_B is effectively positive. Under these conditions the angle of attack is fixed and corresponds to the attitude of the airplane on the ground. We shall call this the "ground angle of attack."

Assuming μ and f to be fixed, equation (1) (on eliminating N_B and N_R, determined by equations (2) and (3) may be written

$$\frac{P}{g}\frac{dV}{dt} = \tag{4}$$

$$F - R_x - \frac{(Fd - R\delta)\ (f - \mu)}{L + (f - \mu)\ H} - (P - R_z)\ \frac{fa + \mu b}{L + (f - \mu)\ H}.$$

The airplane loses momentum on the ground in proportion as the absolute value of the negative dV/dt continues to increase.

It is important, therefore, to give the term

$$\frac{(Fd - R\delta)\ (f - \mu)}{L + (f - \mu)\ H}$$

the greatest possible positive value. In fact, this term is of little importance, because d and δ are both small and $f - \mu$ is a very small factor. In order to simplify the problem, we shall neglect this unimportant term (which, moreover, approaches zero with the speed V, if the propeller does not continuously act as a brake with a positive lever arm d). Equation (4) then becomes

$$\frac{P}{g}\frac{dV}{dt} = - F - R_x - (P - R_z)\ \frac{fa + \mu b}{L + (f - \mu)\ H} \tag{5}$$

or, by putting

$$\frac{fa + \mu b}{L + (f - \mu)\ H} = \psi \tag{6}$$

and designating by c_{x_b} and c_{z_b} the aerodynamic coefficients for the ground angle of attack,

$$\frac{P}{g}\frac{dV}{dt} = -(F + \psi P) - (c_{x_b} - \psi c_{z_b}) \frac{\rho}{2} S V^2 \qquad (7)$$

For simplicity, let us assume that the force F is constant or that F denotes the mean value of the braking force of the propeller during the landing run. Let D represent the length of the run. The speed V equals dD/dt. Replacing dt by dD/V in equation (7) and integrating, we obtain, for the landing run,

$$D = \frac{P/S}{\rho g\ (c_{x_b} - \psi c_{z_b})} \log \frac{F + \psi P + \frac{\rho}{2} S V_2^2\ (c_{x_b} - \psi c_{z_b})}{F + \psi P}$$

or, by introducing the lift c_{z_2} corresponding to the landing speed V_2 and designating the ratio F/P by X,

$$D = \frac{P/S}{\rho g\ (c_{x_b} - \psi c_{z_b})} \log \left[1 + \frac{c_{x_b} - \psi c_{z_b}}{(X + \psi)\ c_{z_2}}\right] \qquad (8)$$

In this formula, c_{z_b} is evidently smaller than or at most, equal to c_{z_2}, since this lift is assumed to be equal to the maximum.

It is easy to show that D is a constantly decreasing function of the difference $c_{x_b} - \psi c_{z_b}$, which it is consequently important to make as large as possible, in order to reduce D as much as possible. Thus, if ψ is fixed, it is important to have, at the ground angle of attack, a strong drag c_{x_b} and a small lift c_{z_b}. In general, however, c_{z_b} is also increased by increasing c_{x_b}.

On ordinary airplanes the ground angle of attack is determined beforehand, and the above condition can be satisfied only by the addition of supplementary drag, which is purely structural or even detrimental to the lift, showing that air-braking while taxying requires aerodynamic characteristics partially at variance with those required for air braking before contact with the ground, for reducing the landing speed.

Lastly, equation (8) renders it possible to investigate the variation in the length D of the landing run with ψ, that is, with the braking coefficient on the ground. Without entering into any intricate mathematical discussion, we will simply state that, whatever may be the values of c_{z_a}, c_{x_b}, c c_{z_b} and X, it is always important to make ψ positive and not too small. It is not certain in advance, however, that it is always important to make ψ as large as possible. Calculation at least renders it possible to determine this definitely in each particular case.

The coefficient ψ, as defined by equation (6), is a sort of fictitious or over-all coefficient of braking on the ground, which simultaneously includes the frictional effect of the ground on the wheels (μ) and on the skid (f) and takes into account the position of the center of gravity with reference to the points of contact A and B of these parts with the ground. For example, we may take $f = 0.4$ for a yielding skid on an average soil.

For skids which take hold more strongly, very high values of f may be attained, but it would seem that such skids should be prohibited, in order to avoid tearing up the landing fields. A special brake might be mounted on the plow of the skid for occasional use in cases of emergency.

On well-kept fields and for ball-bearing brakeless wheels, μ may have values between 0.04 and 0.1. For wheels with drum brakes similar to the ones used on automobiles, μ may be as high as 0.3. On bad ground these figures may be increased, but the taxying then becomes irregular and the risk of capsizing too great for the theoretical calculation of the landing run to be of any practical importance.

To give an idea of the possible values of ψ, we will take the example of an airplane for which

$$a = 0.82 \text{ m}; \quad b = 5.6 \text{ m}; \quad l = a + b = 6.42 \text{ m};$$

$$H = 1.7 \text{ m}$$

Assuming the wheels to be brakeless and taking $f = 0.4$ and $\mu = 0.04$, we have $\psi = \psi_1 = 0.078$. Then assuming the wheels to be braked to the maximum and taking $f = 0.4$ and $\mu = 0.3$, we have $\psi = \psi_2 = 0.306$. From this example we see how the participation of the tail skid in the braking on the ground renders the variation of the over-all coefficient of friction ψ different from that of the coefficient μ of the wheels and is only modified by the braking of the latter.

If the two above-calculated values of ψ (with and without wheel brakes) are applied to the calculation of the landing run of the airplane already considered, we obtain, on assuming the ground angle of attack to be equal to that of the maximum lift ($c_{z_b} = c_{z_a}$), the following results:

TABLE VI.

Cases in Tables IV and V	Landing run in meters	
	Without wheel brakes ψ_1=0.078	With wheel brakes ψ_2=0.306
Case a	340	166
" b	300	89
" c (braking force of propeller is assumed to be P/5)	106	80
Case d	104	77
" e	170	89
" f	54	51

On this table, which is given only by way of illustration, we see how the landing run is shortened by the wheel brakes. This device obviously loses some of its importance, when we have other powerful means of braking, like the one considered in Case c (propeller with very strong braking effect). It is likewise obvious that the devices for amplifying the polar, such as the slotted wing (Case e), considerably reduce the landing run. With the latter device, however, the wheel brakes retain all their importance.

Summary and Conclusions

In the numerical examples, we have considered an airplane landing in calm air in a fixed direction after crossing the border (with its obstacles) at a height of 30 m (98 ft.). Its stopping point is at a distance D from the obstacle, comprising:

A distance D_1 in regular gliding flight;

A distance D_2 in levelling off;

A distance D_3 in taxying on the ground.

The calculations enable us to make out the following table, which gives an idea of the improvements to be expected in the use of the various possible methods of braking in the air and on the ground.

TABLE VII

Modifications made in the airplane		Horizontal distances traversed (in meters) after clearing obstacle 30 m high. D_1	D_2	D_3 Without wheel brakes	D_3 With wheel brakes
(A) Braking in the air	a) Reference airplane	210	180	340	166
	b) Add passive drag (Δc_x=0.04) doubling minimum drag of airplane	140	143	300	89
	c) Add braking propeller in flight (Δc_x=0.16) exerting on the ground, a braking force of P/5	70	113	106	80
	d) Add b + c	60	110	104	77
(B) Amplification of the polar	e) Transform polar by homothety in the ratio λ=2	255	90	170	89
A + B	f) Add b + c + e	73	55	54	51

TABLE VII (Cont.)

Modifications made in the airplane		Horizontal distances traversed (in meters) after clearing obstacle 30 m high. Total distance D: Without wheel brakes	With wheel brakes
(A) Braking in the air	a) Reference airplane	730	556
	b) Add passive drag (Δc_x=0.04) doubling minimum drag of airplane	583	372
	c) Add braking propeller in flight (Δc_x=0.16) exerting on the ground, a braking force of P/5	289	263
	d) Add b + c	274	247
(B) Amplification of the polar	e) Transform polar by homothety in the ratio λ=2	515	434
A + B	f) Add b + c + e	182	179

In the above landing problem, it is important to note that we assumed a very gentle levelling off, i.e., with a very moderate centripetal acceleration. On the other hand, we assumed that the landing field was a good one for taxying. The calculated distances D accordingly represent values near the maximum for the various cases.

In Table VII, we notice that the methods (A) of braking in the air are of considerable importance for landing on a small field surrounded by high obstacles.

Case e, resulting from a hypothesis ($\lambda = 2$) very favorable to the method (B) of the amplification of the polar curve, shows that this method, under the preceding ratio and despite the reduction effected in the landing speed, is decidedly inferior to the methods under (A), namely, Case b, with wheel brakes, and Case c and even Case d.

Moreover, it is important to arrange the braking surfaces or propellers in such manner as to furnish a lifting component and, in general, to combine judiciously the various methods in such a way as to derive the maximum efficacy from each.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

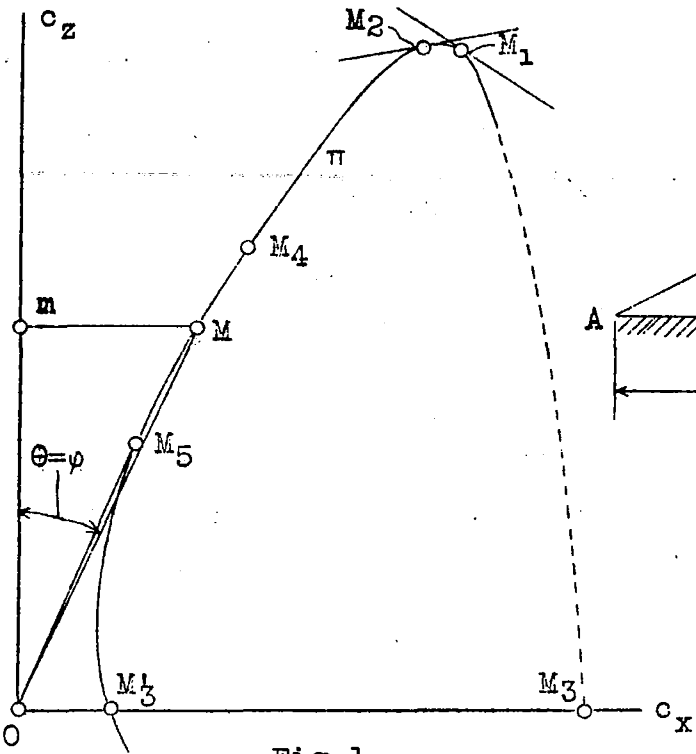


Fig.1

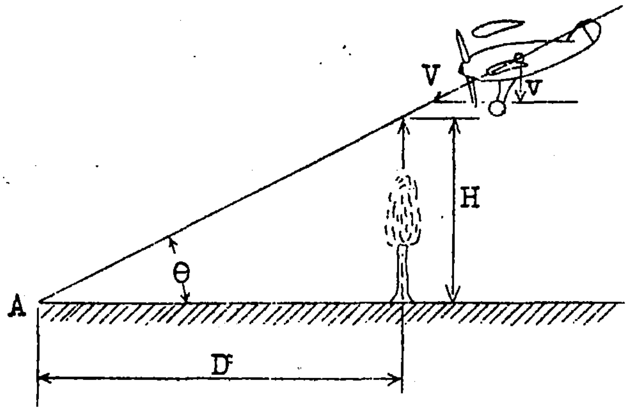


Fig.2

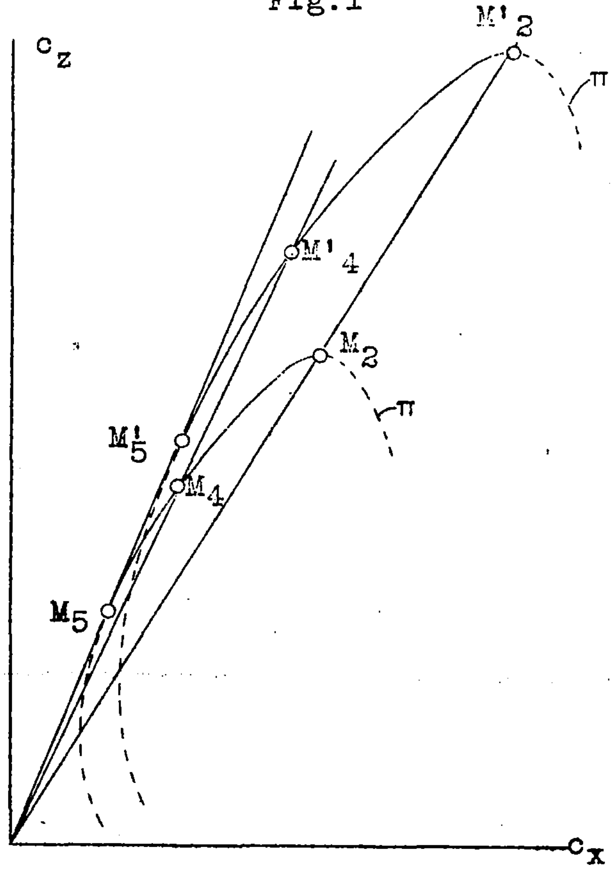


Fig.3

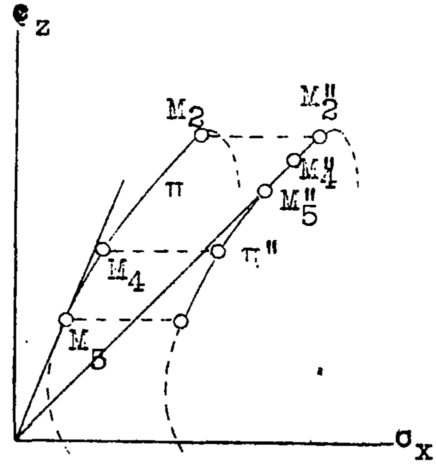
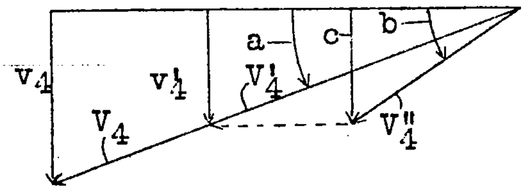


Fig.4

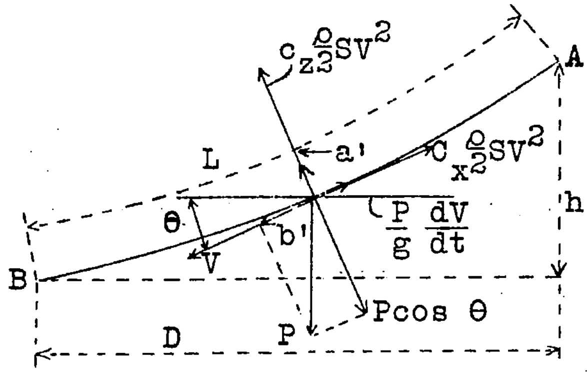


a, $\tan \theta_4 = \tan \theta_4'$

b, $\tan \theta_4''$

c, v_4''

Fig.5



a', $\frac{P}{g} \frac{V^2}{R}$

b', $P \sin \theta$

Fig.6

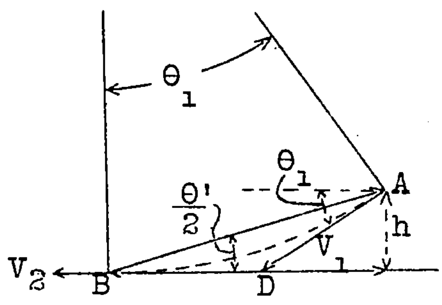


Fig.7

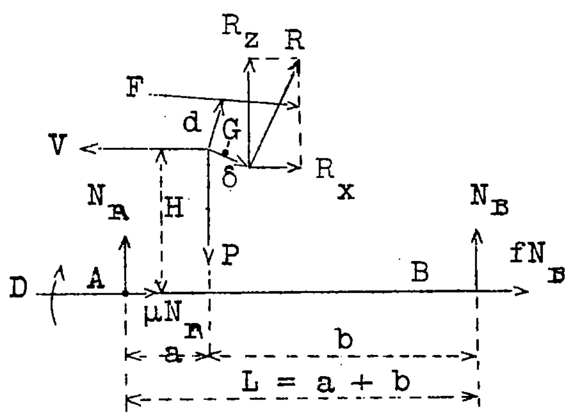


Fig.8